Effect of Sensor and Actuator Errors on Static Shape Control for Large Space Structures

Raphael T. Haftka*
Virginia Polytechnic Institute and State University, Blacksburg, Virginia
and
Howard M. Adelman†
NASA Langley Research Center, Hampton, Virginia

An analytical study using applied temperatures was performed to predict and assess the effect of actuator and sensor errors on the performance of a shape control procedure for flexible space structures. Approximate formulas were derived for the expected value and variance of the rms distortion ratio (ratio of rms distortions with and without corrections) based on the assumption of zero-mean normally distributed random errors in measured distortions and actuator output temperatures. Studies were carried out for a 55 m radiometer antenna reflector distorted from its ideal parabolic shape by nonuniform orbital heating. The first study consisted of varying the sensor and actuator errors and computing the distortion ratio for the case of 12 actuators. In the second study, sensor and actuator errors were prescribed and effect of increasing the number of actuators was evaluated.

Nomenclature

A = matrix defined by Eq. (5)
a_{ij} = elements of matrix A
B = normalizing matrix, Eq. (9)
C_{TT} = covariance matrix of $T_a - T_m$
C_{TT_a} = covariance matrix of $T_a - T_0$
C_{TT_m} = covariance matrix of $T_m - T_0$
C_{VV} = covariance matrix of V
$C_{\psi\psi}$ = covariance matrix of ψ
E = expected value (average) operator
g = distortion ratio, Eq. (7)
g_0 = distortion ratio in the absence of errors
G = matrix defined by Eq. (19)
k = number of structural degrees of freedom
L = the Cholesky factor of B ($B = LL^T$)
n = number of actuators
r = vector defined by Eq. (6)
r_j = jth component of r
r_0 = r vector in the absence of sensor errors
r_m = r vector based on measured distortion
T = vector of temperature differentials
T_a = actuator temperature differential in the presence of errors
T_i = temperature differential of the ith actuator
T_m = actuator temperature differential based on measurement errors
T_{max} = maximum actuator temperature differential
T_0 = actuator temperature differential in the absence of errors
U = matrix of u_i
u_i = structural displacement due to unit temperature increase in ith actuator
v_0 = reference volume
δ = corrected displacement field
δ_{rms} = root mean square of displacement field
σ = standard deviation
σ_T = standard deviation of actuator error
σ_ψ = standard deviation of error in measured distortion
ψ = shape distortion vector
ψ_m = measured shape distortion vector
ψ_{rms} = $(\psi^T B \psi)^{1/2}$
ψ_{max} = maximum component ψ
Ω = structural domain

Introduction

ONE of the most important requirements in the design of large space antennas is that of surface accuracy.[1,2] While studies have shown that under certain conditions high surface accuracies may be maintained by passive means,[3] as a general rule, active controls probably will be needed. The disturbances that deform the shape of space structures can be divided into two types. One type is transient and, hence, eventually leaves the structure unchanged. Transient disturbances can be countered by active or passive controls that enhance the damping of the structure. The second type of disturbance is typical of those due to manufacturing errors and aging,[4] and may be considered at least quasistatic if not fixed. These latter disturbances may be offset by slowly applied, long-acting corrections. Most research to date has concentrated on controlling the transient disturbances by using damping actuators.[5]

Interest in the problem of controlling quasistatic disturbances has recently increased. Much of the work reported on active control of quasistatic disturbances is related to active control of optical systems such as mirrors (see Ref. 6 for a survey of the state-of-the-art as of 1978). Generally, force actuators are employed for static shape control.[7-11] However, Bushnell[7] characterizes some of these[12,13] as displacement actuators because they are stiff enough to enforce prescribed displacements. Another variation of the force actuator in a truss structure is one that changes the length of a member by reeling a cable in or out of by using a screw mechanism. This approach is used to correct fabrication errors on some antennas, albeit on the ground rather than in orbit. A recently proposed alternative is the use of applied temperatures on the structure. In Haftka and Adelman,[15] the basic concept of using heater actuators was described and

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*Professor, Department of Aerospace and Ocean Engineering. Member AIAA.
†Senior Research Engineer, Interdisciplinary Research Office. Member AIAA.

evaluated on beam and antenna examples. In Haftka and Adelman,[16] the problem of finding near-optimum actuator locations was investigated and two methods based on heuristic integer programming were described and evaluated. To date no attempt has been made to study the effects of sensor and actuator errors on static shape control. The purpose of the present paper is to extend the work described in Refs. 15 and 16 to a study of the effect of such errors and to obtain the statistical properties of the control effectiveness as a function of the magnitude of the errors. The analysis is limited to normally distributed errors and to configurations with a large number of sensors. A 55 m space-truss parabolic antenna is used to demonstrate the effects of sensor and actuator errors.

Review of Static Shape Control Method Based on Heater Actuators

Although discussed in the context of heater actuators, the methods presented in this paper are applicable to both linear force and temperature controls. The equations for temperature control from Ref. 15 are briefly summarized herein. (The reader is referred to Ref. 15 for a similar derivation for force controls.)

The structure is assumed to be in Earth orbit and to possess rigid body degrees of freedom. It is defined over some region Ω and its desired shape has presumably been distorted by an amount $\psi(Q)$, where Q is a point in Ω and ψ is a vector containing displacement components in three orthogonal directions. The distortion is corrected by prescribing temperatures at n high-thermal-expansion inserts (actuators) placed in the structure. The distortion ψ is assumed to be slowly varying so that the actuator inputs may be calculated by a quasistatic analysis.

The residual displacement δ is the sum of the shape distortion and the correction

$$\delta(Q)=\psi(Q)+\sum_{i=1}^{n}u_i(Q)T_i \tag{1}$$

where T_i is the temperature differential of the ith actuator relative to the temperature at which ψ is measured, and u_i is the displacment field due to a unit value of T_i.

We seek values of T_i that most effectively offset ψ, that is, cause δ to be close to zero. A common measure of the smallness of δ is based on the rms value

$$\delta^2_{\rm rms}=\frac{1}{v_0}\int_\Omega \delta\cdot\delta\,\mathrm{d}\Omega \tag{2}$$

where v_0 is a reference volume, and a dot represents a scalar product. The necessary condition for a minimum is

$$\frac{\partial\delta^2_{\rm rms}}{\partial T_j}=(2/v_0)\int_\Omega\left(\psi+\sum_{i=1}^{n}u_iT_i\right)\cdot u_j\mathrm{d}\Omega=0 \qquad j=1,2,\ldots,n \tag{3}$$

Equation (3), a system of n linear algebraic equations for the control temperatures, can be written as

$$AT=r \tag{4}$$

where the components a_{ij} of the matrix A is

$$a_{ij}=\int_\Omega u_i\cdot u_j\mathrm{d}\Omega \tag{5}$$

and the jth component of the right-hand side r_j is

$$r_j=-\int_\Omega \psi\cdot u_j\mathrm{d}\Omega \tag{6}$$

The ratio of controlled to uncontrolled rms distortion is called the distortion ratio g and is given by

$$g^2=\int_\Omega\delta\cdot\delta\mathrm{d}\Omega\Big/\int_\Omega\psi\cdot\psi\mathrm{d}\Omega \tag{7}$$

When the displacement vector is computed from a discretized model of the structure having k degrees of freedom, ψ, u, and δ are finite-dimensional vectors of order k instead of vector functions. Equation (1) may then be written in a matrix form as

$$\delta=\psi+UT \tag{8}$$

where U is a matrix whose columns are u_i, $i=1,2,\ldots,n$. Equation (2) becomes

$$\delta^2_{\rm rms}=(\psi+UT)^TB(\psi+UT) \tag{9}$$

where B is a symmetric positive definite matrix. Similarly, the expressions for A, r, and g^2 become

$$A=U^TBU \tag{10}$$

$$r=-U^TB\psi \tag{11}$$

$$g^2=\frac{(\psi+UT)^TB(\psi+UT)}{\psi^TB\psi}=1+\frac{2\psi^TBUT+T^TU^TBUT}{\psi^TB\psi} \tag{12}$$

where use has been made of the symmetry of B.

Effect of Sensor and Actuator Errors

General Equations

We define T_0 as the vector of control temperatures that would be produced in the absence of both sensor and actuator errors. That is, T_0 is calculated from Eq. (4) in terms of the actual distortion ψ by

$$AT_0=r_0=-U^TB\psi \tag{13}$$

Because of sensor error we measure ψ_m instead of ψ and the control algorithm calculates a corresponding temperature vector T_m as

$$AT_m=r_m=-U^TB\psi_m \tag{14}$$

Additionally, there are actuator errors (possibly including computation errors) so that the control output is T_a. The distortion ratio g is then given from Eq. (12) as

$$g^2=1+(2\psi^TBUT_a+T_a^TU^TBUT_a)/\psi^TB\psi \tag{15}$$

Using Eqs. (10) and (13) and the identity $T_a=T_0+(T_a-T_0)$, Eq. (15) may be written as

$$g^2=g_0^2+[(T_a-T_0)^TA(T_a-T_0)]/\psi^TB\psi \tag{16}$$

where g_0 is the ideal distortion ratio (obtained when there are no errors)

$$g_0^2=1+\frac{2\psi^TBUT_0+T_0^TU^TBUT_0}{\psi^TB\psi}=1-\frac{r_0^TT_0}{\psi^TB\psi} \tag{17}$$

Statistical Properties of Distortion Correction

We assume that there are enough sensors to obtain an accurate reading of ψ in the absence of errors and that they have no bias. Then $\psi-\psi_m$ is a random vector with zero mean. Similarly, we assume that there is no actuator bias so

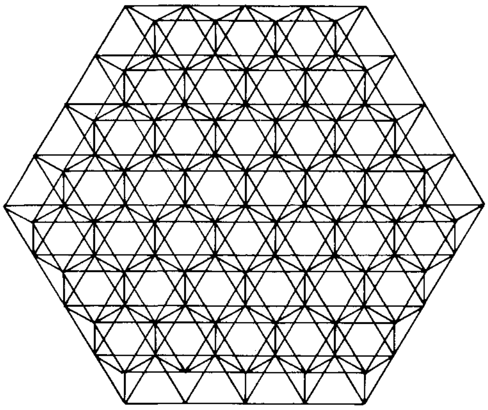

Fig. 1 Tetrahedral truss antenna reflector.

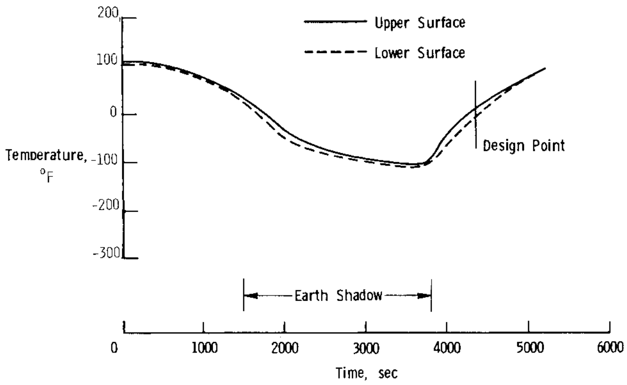

Fig. 2 Temperature history for antenna reflector.

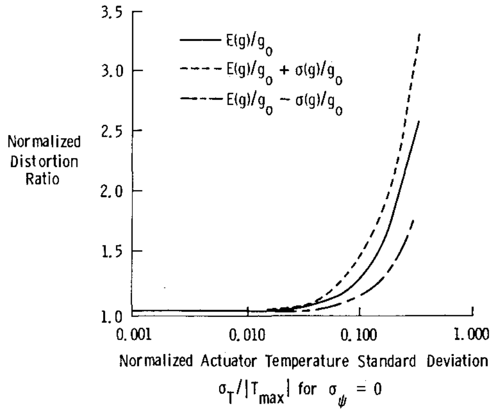

Fig. 3 Effect of actuator error on distortion ratio.

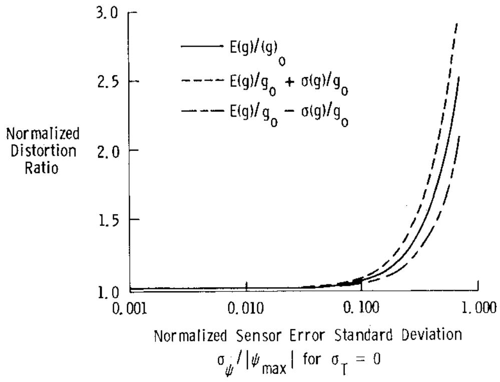

Fig. 4 Effect of sensor error on distortion ratio.

that $T_a - T_m$ is also a random vector with zero mean. Using Eqs. (13) and (14), we obtain

$$T_m - T_0 = G(\psi_m - \psi) \tag{18}$$

where

$$G = -A^{-1}U^T B \tag{19}$$

We further assume that $\psi_m - \psi$ and $T_a - T_m$ are independent, normally distributed vectors with covariance matrices $C_{\psi\psi}$ and C_{TT}, respectively. Based on Eq. (18), $T_m - T_0$ is also a normally distributed random vector and its covariance matrix C_{TT_m} is

$$C_{TT_m} = E[(T_m - T_0)(T_m - T_0)^T] = GC_{\psi\psi}G^T \tag{20}$$

where E denotes the expected value operator. The total control error $T_a - T_0$ may be written as

$$T_a - T_0 = (T_a - T_m) + (T_m - T_0) \tag{21}$$

and is, therefore, the sum of two independent normally distributed zero-mean vectors. Hence, $T_a - T_0$ is also a normally distributed vector with zero mean. The covariance matrix of $T_a - T_0$ is defined as C_{TT_a}. From Eq. (21)

$$C_{TT_a} = C_{TT} + C_{TT_m} \tag{22}$$

Next we obtain the expected value and standard deviation of g^2. For this purpose we define the vector V as

$$V = L^T U(T_a - T_0)/(\psi^T B \psi)^{1/2} \tag{23}$$

where L is the Cholesky factor of B (i.e., $B = LL^T$). Equation (16) may now be written in terms of V as

$$g^2 = g_0^2 + V^T V \tag{24}$$

The expected value of g^2, denoted $E(g^2)$, and the standard deviation $\sigma(g^2)$ may be given in terms of the covariance matrix C_{VV} of V [which, from Eq. (23), is also a normally distributed vector with zero mean]:

$$C_{VV} = L^T U C_{TT_a} U^T L / \psi^T B \psi \tag{25}$$

From Eq. (24), denoting the components of V as v_i,

$$E(g^2) = g_0^2 + \sum_{i=1}^{k} \sigma^2(v_i) \tag{26}$$

where $\sigma^2(v_i)$ (i.e., the variances of the components of V) are the diagonal terms of C_{VV}

$$\sigma^2(v_i) = (C_{VV})_{ii} \tag{27}$$

The variance of g^2 is

$$\begin{aligned}\sigma^2(g^2) &= E[g^2 - E(g^2)]^2 = E\left[V^T V - \sum_{i=1}^{k} \sigma^2(v_i)\right]^2 \\ &= E[(V^T V)^2] - \left[\sum_{i=1}^{k} \sigma^2(v_i)\right]^2 \\ &= \sum_{i=1}^{k}\sum_{j=1}^{k} E(v_i^2 v_j^2) - \left[\sum_{i=1}^{k} \sigma^2(v_i)\right]^2 \end{aligned} \tag{28}$$

Table 1 Effect of sensor and actuator errors on distortion reduction[a]

a) $\sigma_\psi = 10^{-5}$ in., $\sigma_T = 0.01°F$

No. of actuators	g_0	$E(g)$	$\sigma(g)$	$E(g)+\sigma(g)$	$E(g)-\sigma(g)$
10	0.3124	0.3358	0.015	0.3508	0.3208
20	0.1980	0.2645	0.035	0.2995	0.2295
30	0.1793	0.2683	0.043	0.3113	0.2253
40	0.1533	0.2722	0.051	0.3232	0.2212
50	0.1335	0.2790	0.061	0.3400	0.2180
60	0.1120	0.2799	0.069	0.3489	0.2109

b) $\sigma_\psi = 10^{-5}$ in., $\sigma_T = 0.005°F$

No. of actuators	g_0	$E(g)$	$\sigma(g)$	$E(g)+\sigma(g)$	$E(g)-\sigma(g)$
10	0.3124	0.3185	0.0039	0.3224	0.3146
20	0.1980	0.2171	0.011	0.2281	0.2061
30	0.1793	0.2061	0.014	0.2201	0.1921
40	0.1533	0.1913	0.018	0.2093	0.1733
50	0.1335	0.1826	0.024	0.2066	0.1586
60	0.1120	0.1720	0.029	0.2010	0.1430

[a] g_0 = distortion ratio neglecting both errors. σ = one standard deviation.

It is shown in Ref. 17 that

$$E(v_i^2 v_j^2) = (C_{VV})_{ii}(C_{VV})_{jj} + 2(C_{VV})_{ij}^2 \quad (29)$$

using Eqs. (27) and (29) in Eq. (28),

$$\sigma^2(g^2) = 2\sum_{i=1}^{k}\sum_{j=1}^{k}(C_{VV})_{ij}^2 \quad (30)$$

While exact expressions are available for the average and standard deviation of g^2, only approximate expressions are obtained for the corresponding properties of g. Following the method used in Ref. 18 the first three terms of a Taylor series are used. Specifically, for the square root function

$$x^{1/2} \cong x_0^{1/2} + 1/2x_0^{-1/2}(x-x_0) - 1/8x_0^{-3/2}(x-x_0)^2 \quad (31)$$

and substituting g^2 for x and $E(g^2)$ for x_0, we obtain

$$g \cong E^{1/2}(g^2) + 1/2[g^2 - E(g^2)]/E^{1/2}(g^2) - 1/8[g^2 - E(g^2)]^2/E^{3/2}(g^2) \quad (32)$$

Equation (32) is a good approximation if $|g^2 - E(g^2)|$ is small compared to $E(g^2)$. Taking the expected value of both sides of Eq. (32) gives

$$E(g) \cong E^{1/2}(g^2) - [\sigma^2(g^2)/8E^{3/2}(g^2)] \quad (33)$$

$$\sigma^2(g) = E[g - E(g)]^2 = E(g^2) - E^2(g) \cong [\sigma^2(g^2)/4E(g^2)] - [\sigma^4(g^2)/64E^3(g^2)] \quad (34)$$

Antenna Reflector Example

An example of a 55 m space-truss parabolic antenna reflector from Garrett[19] is shown in Fig. 1 and demonstrates the effects of actuator and sensor errors on the distortion ratio. The antenna is constructed of graphite epoxy; the control elements are made of aluminum. The reflector is distorted from its ideal shape by thermal deformation caused by orbital heating. The temperature history of the lower and upper surfaces of the antenna (calculated from Ref. 15) is shown in Fig. 2. A previous study[16] obtained near-optimal actuator locations for controlling the thermal distortion with a varying number of actuators. These actuator locations were used in the present study, and it it was assumed that a sensor is located at each of the joints of the truss so that in the absence of sensor errors all the displacement components at the joints are known. The matrix $\boldsymbol{B}$ of Eq. (9) was taken to be a diagonal matrix with unit entries corresponding to the displacement components normal to the reflector surface. This means that the mission of the control system was to minimize the rms value of the normal displacements at the

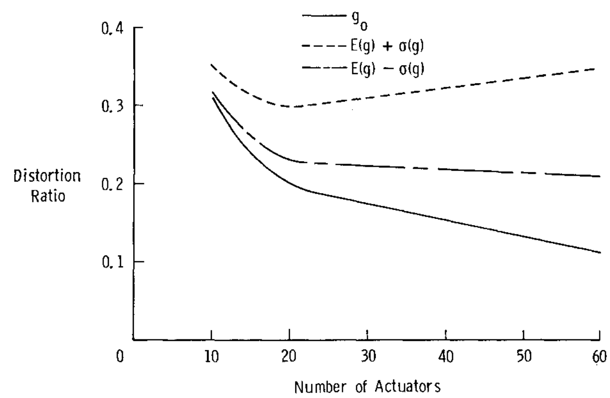

Fig. 5 Effect of sensor and actuator errors on distortion ratio: g_0 = distortion ratio without error ($\sigma_\psi = \sigma_T = 0$); σ = one standard deviation based on $\sigma_\psi = 10^{-5}$ in., $\sigma_T = 10^{-2}$°F.

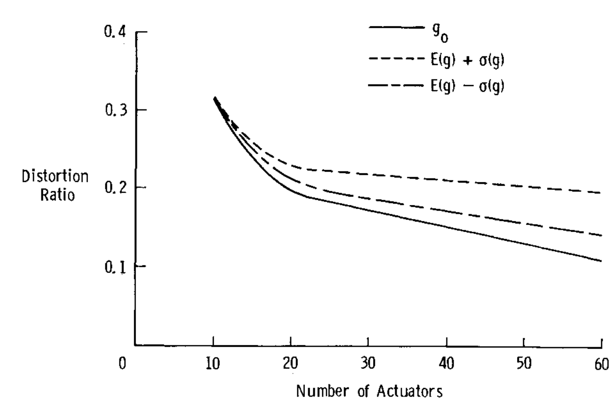

Fig. 6 Effect of sensor and actuator on distortion ratio: g_0 = distortion ratio without error ($\sigma_\psi \sigma_T = 0$); σ = one standard deviation based on $\sigma_\psi = 10^{-5}$ in., $\sigma_T = 5\times10^{-3}$°F.

joints. It was assumed that sensor errors are uncorrelated and all have the same standard deviation σ_ψ. Similarly, it was assumed that actuator errors were uncorrelated and have the same standard deviation σ_T.

First, the effects of sensor and actuator errors were studied for a 12-actuator configuration (from Ref. 16 where $g_0 = 0.275$). The effect of actuator errors on the expected value and standard deviation of the distortion ratio is shown in Fig. 3. In Fig. 3, the actuator temperature error is normalized to the maximum component of the temperature vector $\boldsymbol{T}_0$. The distortion ratio is normalized to g_0. As may be predicted on the basis of Eqs. (26) and (30), $E(g)$ and $\sigma(g)$ vary quadratically with σ_T. This is due to the actuator temperatures, which are the solution of a minimization problem so that small errors in the temperatures do not have any first order effect. However, it is seen from Fig. 3 that errors with σ_T greater than about 10% of the maximum actuator temperature differential substantially degrade the effectiveness of the control procedure. The bands of width $\sigma(g)/g_0$ indicate the scatter of the distortion ratio, but since g is not a normal random variable, care must be exercised in the interpretation of $\sigma(g)$. Figure 4 shows the effect of sensor error on the expected values and standard deviation of the distortion ratio. In Fig. 4, the sensor error is normalized to the maximum component of the distortion vector ψ. Here, substantial degradation in performance occurs for errors that are above 10% of the maximum component of the distortion vector.

Next, the effect of the number of actuators was studied. An error combination that has a moderate effect on the 12-actuator configuration was selected. The value of σ_ψ was selected to be 10^{-5} in. (corresponding to 0.25% of the maximum component of ψ), and the value of σ_T was selected to be 0.01 deg (corresponding to 6.5% of the 12 actuator maximum temperature differential). This combination corresponds to $E(g)/g_0$ of 1.11 for the 12-actuator configuration. The number of actuators was varied from 12 to 60, with the same values of σ_T and σ_ψ. The results are summarized in Table 1a and Fig. 5, which show the one-σ scatter band about $E(g)$. Figure 5 shows that as the number of actuators increases, the effect of the error becomes more severe—to the point that the benefit of increasing the number of actuators beyond twenty is questionable.

The same study was repeated for a smaller error combination. The results are summarized in Table 1b and Fig. 6. While it is clear that the effect of the error becomes more severe as the number of actuators increases, the error is small and the performance still improves.

Concluding Remarks

An analytical study using applied temperatures was performed to predict and assess the effect of actuator and sensor errors on the performance of a shape control procedure for flexible space structures. Approximate formulas were derived for the expected value and variance of the rms distortion ratio (ratio of rms distortions with and without corrections) based on the assumption of zero-mean normally distributed random errors in measured distortions and actuator output temperatures.

Studies were carried out for a 55 m radiometer antenna reflector that was distorted from its ideal parabolic shape by nonuniform orbital heating. The first study consisted of varying the sensor and actuator errors and computing the distortion ratio for the case of 12 actuators. It was found that substantial inaccuracies occurred when the actuator error exceed about 10% of the maximum actuator temperatures differential, or when the sensor error exceeded about 10% of the maximum component of distortion. In a second study, sensor and actuator errors were held constant and the number of actuators was varied. Results of this study indicated that for sufficiently large errors, the error magnification, due to increasing the number of actuators, can offset the benefits of having more actuators. This leads to the anomalous result that increasing the number of actuators can eventually degrade the performance of the control system.

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